

Brane Embeddings in $AdS_4 \times \mathcal{CP}^3$

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Abstract

We construct D -brane embeddings in $AdS_4 \times \mathcal{CP}^3$ by studying the consistency conditions following from the pull back of target space equations of motion. We explicitly discuss the supersymmetry preserved by these embeddings by analyzing the compatibility of kappa symmetry projections with the target space Killing spinors in each case. The embeddings correspond to AdS/dCFT dualities involving a CFT with a defect.

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I. INTRODUCTION

Recently, string theory on $AdS_4 \times \mathcal{CP}^3$ has enjoyed a special study, due to its appearance in a new example of the AdS/CFT duality [1]. The example involves, $\mathcal{N} = 6$ superconformal $SU(N) \times SU(N)$ Chern-Simons theory in three dimensions and M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$, where k is the level of the Chern-Simons action [2]. The new duality is motivated by the conjecture that Superconformal Chern-Simons theories describe the low-energy world-volume dynamics of multiple M2-branes [3]. The ABJM model is characterized by two parameters – the rank N of the two gauge groups $SU(N)$ and the integer level k which is opposite for the gauge groups. Remarkably, there exists an analogue of the 't Hooft limit, where $N, k \rightarrow \infty$ with the ratio $\lambda = 2\pi^2 N/k$ kept fixed. In this limit λ becomes continuous allowing therefore for application of standard perturbative techniques. In particular [2], at strong coupling, i.e, when λ becomes large, the M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ can be effectively described by type IIA superstring theory on the $AdS_4 \times \mathcal{CP}^3$ background. The connection between M-theory and type IIA string theory has been an exciting source of information, especially, since the fundamental strings and D-branes of type IIA are on a unified footing in eleven dimensions. Consequently, starting from M-theory, it is possible to have an alternative description as a weakly coupled type IIA string theory, giving us the handle to bridge the non-perturbative and perturbative features of these theories. Low energy theory on the M2-branes governed by non-dynamical CS-theory can have interesting relation to the dynamical Yang-Mills living on the world-volume of D -branes in type IIA [4, 5]. Construction of string and brane actions for IIA strings in $AdS_4 \times \mathcal{CP}^3$ is a very interesting subject and has been discussed extensively in [6]-[16].

An intriguing direction to explore for an extension of the new duality [2], is to consider CFT's with a defect, corresponding to D -branes in the bulk. Such theories have several applications in a variety of situations as discussed extensively in the context of $AdS_5 \times S^5$ [17]-[28]. One of the models proposed in [19] corresponds to that of a 3-brane living inside an AdS_5 can give rise to a warped geometry of the form AdS_4 . Such a brane is expected to be a viable candidate for proposals of localized gravity [21]. Interestingly, the metric for such curved branes in AdS can be written down, allowing for a detailed analysis. Thus, in this work, we study the embeddings of D -branes in $AdS_4 \times \mathcal{CP}^3$. These embeddings can actually lead to dualities between supersymmetric AdS embeddings and conformal field theories with a defect, AdS/dCFT duality [18]-[26]. In fact, such situations leading to AdS_4/CFT_3 with flavors have been considered in [29]-[33] and D -branes in various AdS_4 models have been discussed in [34, 35]. In particular, kappa-symmetry gauge fixings of the superstring, $D0$ and $D2$ -brane actions in the complete $AdS_4 \times \mathcal{CP}^3$ superspace have been discussed in [16]. Studying branes in $AdS_4 \times \mathcal{CP}^3$ is also important from the point of view understanding local and non-local operators in the holographic dual theory [36] A discussion of particle like branes was presented in [2], both from the M-theory and IIA point of view. Here, the brane embeddings we discuss are non compact. Furthermore, our discussion of embeddings of branes in $AdS_4 \times \mathcal{CP}^3$ is slightly different from the analysis considered above. We take the lead of [37], where a very general study of possible supersymmetric embeddings of D -branes in $AdS_5 \times S^5$ and its penrose limit, was undertaken. The most general D -brane field equations following from the pull over of the equations of motion in the target space geometry were written down

and explicit embeddings were constructed [37]. This type of analysis is better to understand the classification of embeddings based on the symmetry properties associated to a particular geometry, which is in this case, the $AdS_4 \times \mathcal{CP}^3$ background. The stability analysis involves finding out the number of target space supersymmetries compatible with the kappa symmetry projection on to the brane world volume. This D -brane picture gives a nice view of the dynamics of a defect on the boundary CFT. It is thus interesting to consider an AdS brane in the present context, as it also lends itself to a very natural holographic interpretation. To realize this situation, one adds additional structure on both sides of the duality: a D-brane in the bulk and a defect in the boundary theory. The theory on the defect captures holographically the physics of the D-brane in the bulk, and the interactions between the bulk and the D-brane modes are encoded in the couplings between the boundary and the defect fields.

The new AdS_4/CFT_3 duality shares many common features with the well studied $AdS_5 \times S^5$ -Super Yang-Mills correspondence. In fact, using the insights from the later, many results can be obtained for the former. In tune with the earlier AdS/dCFT dualities [20], in the following, we consider a $D4$ -brane wrapping an $AdS_3 \times \mathcal{CP}^1$ submanifold of $AdS_4 \times \mathcal{CP}^3$. This configuration may be considered as the near-horizon limit of a certain $D2 - D4$ system, and the AdS/CFT duality is considered to act twice: both in the bulk and on the worldvolume. In the limit discussed in [22], the bulk description can be taken to be in terms of supergravity coupled to a probe $D4$ -brane. The dual theory is the ABJM model [2] coupled to a two dimensional defect. The defect theory may be associated with the boundary of AdS_3 and it should be a conformal field theory, following the logic established for earlier brane embeddings [20]. In fact, the existence of solutions we present in this work, gives enough evidence for the existence of dCFT's in the case of AdS_4/CFT_3 duality.

The plan of the rest of the paper is as follows. In Section II, we discuss the D-brane embeddings into the $AdS_4 \times \mathcal{CP}^3$ IIA background. In Section III, we use the kappa symmetry projector to determine the supersymmetry preserved by these D-brane embeddings and conclude in Section IV. Finally, in Appendix A we discuss the D-brane field equations used in Section II.

Note : After submission of this work, we were informed of D -branes and defect ABJM models considered in [29], which has some overlap. More recently, an extensive discussion of branes in $AdS_4 \times \mathcal{CP}^3$, including the construction of defect CFT's appeared in [38].

II. D-BRANE EMBEDDINGS

In this section, we start by giving a heuristic discussion of brane embeddings in $AdS_4 \times \mathcal{CP}^3$ background and present some general features. The issue of stability of the embeddings is dealt with in more detail in the later sections when analyzing the supersymmetry. The $AdS_4 \times \mathcal{CP}^3$ IIA background geometry we start with has the form [2, 39],

$$ds_{IIA}^2 = \tilde{R}^2(ds_{AdS_4}^2 + 4ds_{CP^3}^2), \quad (1)$$

where

$$\tilde{R}^2 = \frac{R^3}{4k} = \pi \sqrt{\frac{2N}{k}}. \quad (2)$$

$$ds_{AdS_4}^2 = \frac{du^2}{u^2} + u^2(dx \cdot dx)_3 \quad (3)$$

$$= \frac{du^2}{u^2} + u^2[-(dx^0)^2 + (dx^1)^2 + (dx^2)^2] \quad (4)$$

$$ds_{CP^3}^2 = d\xi^2 + \cos^2 \xi \sin^2 \xi \left(d\psi + \frac{\cos \theta_1}{2} d\varphi_1 - \frac{\cos \theta_2}{2} d\varphi_2 \right)^2 \\ + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\varphi_2^2). \quad (5)$$

Now, we wish to introduce a $D4$ -brane in this background which wraps an \mathcal{CP}^1 inside the \mathcal{CP}^3 . This can be taken to be defined as:

$$\xi = 0 \quad (6)$$

The radius of the S^2 which the brane wraps is \tilde{R} . The brane world volume lies inside the AdS_4 and stretches along the direction $x^2 = x = 0$. Thus, it fills the AdS_3 defined by the coordinates u, x_0, x_1 and wrapping the \mathcal{CP}^1 parameterized by θ_1, ϕ_1 . The isometry group of the metric (1), preserved by the $D4$ -brane is $SO(2, 2) \times SU(3) \times U(1)$. From a field theory viewpoint $SU(3) \times U(1)$ is the unbroken R-symmetry and $SO(2, 2)$ is the 2D conformal group, suggesting that the dual field theory must be exactly conformal. In the near horizon limit, one expects superconformal enhancement to twelve supercharges.

Using a general ansatz for the embedding surface as:

$$x = \frac{C}{u}, \quad (7)$$

the induced metric can be seen to be AdS_3 :

$$ds^2 = (1 + C^2) \frac{du^2}{u^2} + u^2[-(dx^0)^2 + (dx^1)^2] \quad (8)$$

This leads to a shift in the curvature radius to $r^2 = \tilde{R}^2(1 + C^2)$. Here, C denotes the minimum distance of the AdS_3 brane from the center. This distance is controlled depending on whether some of the $D2$ -branes end on a $D4$ -brane and related the ratio of their tensions [40]. Similar to the various cases of brane embeddings considered in [40], one can check that the situation where q of the N $D2$ -branes end on the $D4$ -brane, lead to a nonzero value for C . This is essentially because, as a reaction to being pulled on by the $D2$ -branes, the $D4$ -brane position along x becomes a function of u . Since this is also a co dimension 3 defect, the bending is of the form given in eqn.(7).

Notice that we are considering the case of an $M5$ -brane in the $AdS_4 \times S^7/\mathbb{Z}_k$ geometry coming from N $M2$ branes at a \mathbb{Z}_k singularity. The intersecting case, where none of the branes are

intersecting corresponding to with a $C = 0$ AdS_3 inside the AdS_4 times an equatorial S^3 inside the S^7 . When there is an additional world-volume flux, the bending was argued in [40] to go as $x = C/u^2$, describing the embedding inside the AdS_4 . It was further pointed out that this behavior is only adequate far away from the intersection and that in the $M2$ near-horizon region the $M5$ -brane bending is different. From the type IIA picture, we see from the above analysis that this is indeed true and the bending is actually as given in eqn.(7). The naive analysis above was all in the probe approximation and it is important to see how the situation changes when the back reaction effects are considered. In particular, the embedded branes are sitting at the top of the potential, signaling a tachyonic instability. However, the mass of the mode is still above the Breitenlohner-Freedman bound [41] and hence is stable.

The DBI part of the action of the embedded D -brane on S^2 , the case where $\xi = 0$ takes the form $\sqrt{1+q^2}$, in the units $\tilde{R} = 1$. Similarly, the contribution of the DBI action from AdS_3 and the WZ-terms can be calculated. The RR 4-form reads

$$F^{(4)} = \frac{3R^3}{8} \epsilon_{AdS_4} \quad (9)$$

$$= \frac{3R^3}{8} u^2 dx^0 \wedge dx^1 \wedge dx^2 \wedge du \quad (10)$$

where ϵ_{AdS_4} is the unit volume form of the AdS_4 space. The RR 2-form $F^{(2)} = d\tilde{A}$ in the type IIA string is explicitly given by

$$F^{(2)} = k(-\cos \xi \sin \xi d\xi \wedge (2d\psi + \cos \theta_1 d\varphi_1 - \cos \theta_2 d\varphi_2) - \frac{1}{2} \cos^2 \xi \sin \theta_1 d\theta_1 \wedge d\varphi_1 - \frac{1}{2} \sin^2 \xi \sin \theta_2 d\theta_2 \wedge d\varphi_2) \quad (11)$$

Using these, we get the full result:

$$\mathcal{L} \approx -T_{D4} \left(R^2 \sqrt{1 + \pi^2 q^2} u \sqrt{u^4 + (u')^2} - q u^3 \right) \quad (12)$$

Both the cases $q = 0$ and $q \neq 0$ exist and the difference is in the AdS curvatures. One can derive the equations of motion and check the validity of solutions. We will not try to do this, as we do it in full generality in the following section. Since the remaining part of spacetime is an AdS_3 , one does not need to be bothered by the tachyonic mode corresponding to fluctuations in S^2 .

Let us now look at the full set of equations of motion specifically for a D4-brane embedding into a $AdS_4 \times \mathcal{CP}^3$ background. Following the general procedure given in [37], for the world volume analysis of Dp -brane field equations in $AdS_n \times S^m$ space times, we discuss the embeddings of branes in CP^3 . A summary of the D-brane field equations is given in Appendix A. Using (A2), the $D4$ -brane field equations in $AdS_4 \times \mathcal{CP}^3$ background, reduces to:

$$e^{-\Phi} \partial_i (\sqrt{-M} \theta^{i i_1}) = \frac{1}{(2!)^2} \epsilon^{i_1 i_2 i_3 i_4 i_5} F_{i_2 i_3} f_{i_4 i_5} + \frac{1}{4!} \epsilon^{i_1 i_2 i_3 i_4 i_5} f_{i_2 i_3 i_4 i_5}. \quad (13)$$

$$\begin{aligned} & \frac{1}{(2!)^3} \epsilon^{i_1 i_2 i_3 i_4 i_5} (F_{i_1 i_2} \wedge F_{i_3 i_4}) f_{i_5 m} + \frac{1}{2!3!} \epsilon^{i_1 i_2 i_3 i_4 i_5} F_{i_1 i_2} f_{i_3 i_4 i_5 m} \\ & = e^{-\Phi} \left[-\partial_i (\sqrt{-M} G^{ij} \partial_j X^n g_{mn}) + \frac{1}{2} \sqrt{-M} (G^{ij} \partial_i X^n \partial_j X^p g_{np, m}) \right]. \end{aligned} \quad (14)$$

where $f_{i_3 i_4 i_5 m}$ denotes the pullback of f on the first three indices, i.e. $f_{i_3 i_4 i_5 m} = \partial_{i_3} X^{m_3} \partial_{i_4} X^{m_4} \partial_{i_5} X^{m_5} f_{m_3 m_4 m_5 m}$ and in a similar manner for $f_{i_5 m}$. The solution set of these equations describes all possible embeddings of D4-branes into the target space. Here we describe D4-branes which wrap on \mathcal{CP}^1 in the \mathcal{CP}^3 . Such embeddings can be realized from the following ansatz: split the embedding coordinates X^m into $\{\eta^i, X^\lambda(\eta^i)\}$, where the worldvolume coordinates are

$$\eta^i = \{x^0, x^1, u, \theta_1, \varphi_1\} \quad (15)$$

and the transverse scalars are

$$X^\lambda = \{x^2(u) \equiv x(u), \xi, \psi, \theta_2, \varphi_2\}, \quad (16)$$

where we relabel x^2 as x and we assume that the only dependence of the transverse scalars on the worldvolume coordinates is in $x(u)$. We switch on a worldvolume flux

$$F_{\theta_1 \varphi_1} = q \sin \theta_1. \quad (17)$$

Notice that this 2-form flux is proportional to the RR 2-form in eqn. (11) at the point $\xi = 0$. With this ansatz, we proceed to calculate all the quantities appearing in (13) and (14); for example,

$$\sqrt{-M} = \tilde{R}^3 u (1 + u^4 (x')^2)^{\frac{1}{2}} L_{(\theta_1 \xi)} \quad (18)$$

where prime denotes the derivative with respect to u and we define,

$$L_{(\theta_1 \xi)} = \left(\tilde{R}^4 \cos^4 \xi (\sin^2 \xi \cos^2 \theta_1 + \sin^2 \theta_1) + q^2 \sin^2 \theta_1 \right)^{\frac{1}{2}}. \quad (19)$$

Substituting the ansatz into (14), we find that the equations derived from the $X^m = \{x^0, x^1, \psi, \theta_2, \varphi_1, \varphi_2\}$ equations are satisfied automatically by the ansatz. And the rest of the equations are discussed below.

The $X^m = x^2 = x$ equation gives :

$$\partial_u \left(\frac{L_{(\theta_1 \xi)}}{(1 + u^4 (x')^2)^{\frac{1}{2}}} u^5 x' - q u^3 \sin \theta_1 \right) = 0; \quad (20)$$

The $X^m = \xi$ equation leads to

$$L_{(\theta_1 \xi)}^{-1} u (1 + u^4 (x')^2)^{\frac{1}{2}} \left[\cos^3 \xi \sin \xi (\cos^2 \xi \cos^2 \theta_1 - 2 \sin^2 \xi \cos^2 \theta_1 - 2 \sin^2 \theta_1) \right] = 0 \quad (21)$$

And the $X^m = \theta_1$ equation leads to

$$L_{(\theta_1 \xi)}^{-3} u (1 + u^4 (x')^2)^{\frac{1}{2}} \cos^4 \xi \sin^2 \xi \cos \theta_1 \sin \theta_1 = 0 \quad (22)$$

Moreover, the gauge field equation (13) leads to :

$$L_{(\theta_1 \xi)}^{-3} u (1 + u^4 (x')^2)^{\frac{1}{2}} \cos^4 \xi \sin^2 \xi \cos \theta_1 = 0 \quad (23)$$

The equation deriving from u follows from the x -equation, and that from θ_1 follows from the gauge field equation. So, for the above ansatz, we find that the only independent equations are

the ones deriving from the $X^m = \{x, \xi\}$ equations along with the gauge field equation (This is expected as worldvolume diffeomorphisms can be used to eliminate $p + 1 = 5$ equations. In addition the metric does not depend upon 3 coordinates $\psi, \varphi_1, \varphi_2$).

Let us summarize the independent equations

$$\partial_u \left(\frac{L_{(\theta_1 \xi)}}{(1 + u^4(x')^2)^{\frac{1}{2}}} u^5 x' - qu^3 \sin \theta_1 \right) = 0; \quad (24)$$

$$\begin{aligned} L_{(\theta_1 \xi)}^{-1} u(1 + u^4(x')^2)^{\frac{1}{2}} \left[\cos^3 \xi \sin \xi (\cos^2 \xi \cos^2 \theta_1 - 2 \sin^2 \xi \cos^2 \theta_1 - 2 \sin^2 \theta_1) \right] &= 0; \\ L_{(\theta_1 \xi)}^{-3} u(1 + u^4(x')^2)^{\frac{1}{2}} \cos^4 \xi \sin^2 \xi \cos \theta_1 &= 0 \end{aligned} \quad (25)$$

To solve (24)-(25) simultaneously, we first note that the ξ -equation (25) and the gauge field equation (25) together can be solved either when (i) $\xi = 0$, which we will refer to as a maximal, as the radius of the CP_1 over which the brane is wrapped $\sim \cos^2 \xi$, or when (ii) $\xi = \frac{\pi}{2}$, which we will refer to as a minimal. The x -equation (24) yields

$$x' = \frac{(qu^3 \sin \theta_1 - c)}{u^2 \sqrt{u^6 L_{(\theta_1 \xi)}^2 - (qu^3 \sin \theta_1 - c)^2}}, \quad (26)$$

where c is an integration constant.

Branes wrapping CP: *Case A*

Let us now substitute solutions of the ξ -equation and gauge field equation into (26). We focus first on the case of the brane wrapping are maximal i.e., the case when $\xi = 0$. In this case (19) simplifies to,

$$L_{(\theta_1 \xi)} = (\tilde{R}^4 + q^2)^{\frac{1}{2}} \sin \theta_1 \quad (27)$$

Using (27), (26) reduces to

$$x' = \frac{(qu^3 - c)}{u^2 (\tilde{R}^4 u^6 + 2cqu^3 - c^2)^{\frac{1}{2}}}, \quad (28)$$

and the induced metric on the brane is

$$ds^2 = \tilde{R}^2 \left[u^2 (dx \cdot dx)_2 + \frac{u^4 (1 + q^2)}{(\tilde{R}^4 u^6 + 2cqu^3 - c^2)} du^2 + (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2) \right], \quad (29)$$

Setting $\tilde{R} = 1$, the induced metric takes a form,

$$ds^2 = u^2 (dx \cdot dx)_2 + \frac{u^4 (1 + q^2)}{(u^3 - u_+^3)(u^3 + u_-^3)} du^2 + (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2), \quad (30)$$

where we write

$$(u^6 + 2cqu^3 - c^2) = (u^3 - u_+^3)(u^3 + u_-^3), \quad (31)$$

with $u_+^3, u_-^3 \geq 0$. The explicit form for the roots of (31) are

$$\begin{aligned} u_+^3 &= -cq + |c| \sqrt{1 + q^2}; \\ u_-^3 &= cq + |c| \sqrt{1 + q^2}. \end{aligned} \quad (32)$$

Now, the $AdS_3 \times S^2$ embeddings can be found as follows. When $c = 0$, (28) integrates to the simple expression

$$x = x_0 - \frac{q}{u}, \quad (33)$$

This solution corresponds to the Karch-Randall embedding [20]. In this limit, the induced metric is,

$$ds^2 = u^2(dx \cdot dx)_2 + \frac{(1+q^2)}{u^2}du^2 + (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2), \quad (34)$$

The embedded geometry is $AdS_3 \times S^2$ when $c = 0$. Also note that the embeddings exist even in the zero flux ($q = 0$) limit. The zero flux embedding must satisfy the zero extrinsic curvature trace condition (A8) since for this solution J_m vanishes.

An explicit calculation of the components of the second fundamental form given in (A7) leads to the following nonzero \mathcal{K}_{ij}^m components while all other components vanish.

$$\begin{aligned} \mathcal{K}_{x^0x^0}^{x^2} &= x'u^3, \mathcal{K}_{x^1x^1}^{x^2} = -x'u^3, \mathcal{K}_{uu}^{x^2} = \frac{-3x'}{u} - x'', \mathcal{K}_{uu}^u = x'^2u^3 \\ \mathcal{K}_{\theta_1\theta_1}^\xi &= \frac{-1}{4} \cos \xi \sin \xi, \mathcal{K}_{\theta_1\varphi_1}^\xi = \frac{1}{4} \cos \xi \sin \xi [\cos^2 \theta_1 \cos 2\xi - \sin^2 \theta_1] \\ \mathcal{K}_{\theta_1\varphi_1}^\psi &= \frac{-1}{4} \cos^2 \xi \sin \theta_1 - \frac{\sin^2 \xi}{2 \sin \theta_1} - \frac{\cos^2 \theta_1}{\sin \theta_1} \cos^2 \xi \\ \mathcal{K}_{\theta_1\varphi_1}^{\varphi_1} &= \cot \theta_1 - \frac{1}{4} [3 + \cos 2\xi] \cot \theta_1, \mathcal{K}_{\varphi_1\varphi_1}^{\theta_1} = \cos \theta_1 \sin \theta_1 [\cos^2 \xi - 1] \end{aligned} \quad (35)$$

For consistency, one can check that the trace of the second fundamental form of the embedding vanishes. Note that when $q = 0$ from (33), x being a constant, those \mathcal{K}_{ij}^m components vanish which are functions of x' or x'' . Again, for the specific embedding using $\xi = 0$, rest of the above listed components vanish except $\mathcal{K}_{\theta_1\varphi_1}^\psi$. Thus, unlike the similar embeddings considered in [37], the ones corresponding to $D4$ -branes wrapping an S^2 in the present context are not totally geodesic [45]. This is due to the marked difference in the sphere and \mathcal{CP}^3 geometries. The second fundamental form is an important quantity when studying the possible corrections to the DBI action at higher orders in α' . Since, the second fundamental form does not vanish, it modifies the pull back of the ambient curvature tensor and contributes to processes involving the scattering of closed and open strings.

We now see how the asymptotically $AdS_3 \times S^2$ embeddings look like. For the general case when $c \neq 0$, the induced geometry is asymptotically $AdS_3 \times S^2$ for $u \gg u_+$. When $u < u_+$, (28) implies that the brane ends at $u = u_+$. To get rid of singularity, changing variables to $u = u_+ + \rho^2$ with $\rho \ll 1$ gives,

$$ds^2 = u_+^2(dx \cdot dx)_2 + \frac{u_+^2(1+q^2)}{(u_+^3 + u_-^3)}d\rho^2 + (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2), \quad (36)$$

In fact, one can make another change of variables to see that the radial coordinate has a range as in [37], indicating a mass gap in the dual theory.

Branes wrapping CP: *Case B*

Let us now discuss embeddings in which the brane wrapping are minimal i.e $\xi = \frac{\pi}{2}$. In this case (26) reduces to

$$x' = \frac{(qu^3 - c)}{u^2(2cqu^3 - c^2)^{\frac{1}{2}}}. \quad (37)$$

It is useful to rescale the parameter c such that $c = Cq$; this removes all q dependence in x' :

$$x' = \frac{(u^3 - C)}{u^2(2Cu^3 - C^2)^{\frac{1}{2}}}. \quad (38)$$

and the induced metric on the brane is then

$$ds^2 = \tilde{R}^2 \left[u^2(dx \cdot dx)_2 + \frac{u^4 du^2}{2C(u^3 - \frac{C}{2})} \right]. \quad (39)$$

Note that x' becomes imaginary for $u^3 < \frac{1}{2}C = u_c^3$, which implies that the brane ends at u_c . The induced geometry is non-singular at u_c and the embedded hyper surface is incomplete.

The induced metric on the \mathcal{CP}^1 is degenerate. In order to interpret such embeddings in which the S^2 is minimal, let us look for D2-brane embedded in the AdS_4 and lie at a point in the \mathcal{CP}^3 . In fact the D4-brane has effectively collapsed to a D2-brane embedded in AdS_4 , which can be verified deriving explicitly the solution of the D2-brane equations of motion. The D2-brane field equations in the $AdS_4 \times \mathcal{CP}^3$ target space are

$$\frac{1}{3!} \epsilon^{i_1 i_2 i_3} f_{i_1 i_2 i_3 m} = e^{-\Phi} \left[-\partial_i (\sqrt{-M} G^{ij} \partial_j X^n g_{mn}) + \frac{1}{2} \sqrt{-M} (G^{ij} \partial_i X^n \partial_j X^p g_{np,m}) \right]. \quad (40)$$

Our ansatz for the worldvolume coordinates is

$$\eta^i = \{x^0, x^1, u\} \quad (41)$$

while the transverse scalars are

$$X^\lambda = \{x^2(u) \equiv x(u), \xi, \psi, \varphi_1, \varphi_2, \theta_1, \theta_2\}, \quad (42)$$

Then the only equation of motion which is not trivially satisfied by the ansatz is

$$\partial_u \left(\frac{u^5 x'}{\sqrt{1 + u^4 (x')^2}} - u^3 \right) = 0. \quad (43)$$

Note that this is precisely the x field equation whereas the u field equation follows from it as the previous case. Since the general solution of (43) is (38), this implies that the D4-brane wrapping a minimal sphere can be interpreted as a D2-brane. When the flux on the D4-brane is positive we get a D2-brane, whereas negative flux corresponds to anti-D2 brane. These embeddings will be shown to break all supersymmetry. A new ansatz describing the D4-branes wrapping the \mathcal{CP}^1 and whose worldvolume lie along x is helpful to preserve supersymmetry. Let us consider,

$$\begin{aligned} \eta^i &= \{x^0, x^1, x, \theta_1, \varphi_1\}; \\ X^\lambda &= \{u, \xi, \psi, \theta_2, \varphi_2\}; \\ F_{\theta_1 \varphi_1} &= q \sin \theta_1. \end{aligned} \quad (44)$$

where all transverse scalars are constant. Now we have :

$$\sqrt{-M} = \tilde{R}^3 u^3 L_{(\theta_1 \xi)} \quad (45)$$

$$L_{(\theta_1 \xi)} = \left(\tilde{R}^4 \cos^4 \xi (\sin^2 \xi \cos^2 \theta_1 + \sin^2 \theta_1) + q^2 \sin^2 \theta_1 \right)^{\frac{1}{2}}. \quad (46)$$

The only field equations which are not already satisfied by the ansatz are

$$\begin{aligned} u : \quad & u^3 (L_{(\theta_1 \xi)} - q \sin \theta_1) = 0; \\ \xi : \quad & L_{(\theta_1 \xi)}^{-1} u^3 \left[\cos^3 \xi \sin \xi (\cos^2 \xi \cos^2 \theta_1 - 2 \sin^2 \xi \cos^2 \theta_1 - 2 \sin^2 \theta_1) \right] = 0; \end{aligned} \quad (47)$$

And the gauge field equation,

$$L_{(\theta_1 \xi)}^{-3} u^3 \cos^4 \xi \sin^2 \xi \cos \theta_1 = 0. \quad (48)$$

The equation deriving from θ_1 follows from the gauge field equation. As before the gauge field equation and the ξ equation can be solved either when (i) $\xi = 0$ or when (ii) $\xi = \frac{\pi}{2}$. In case (i),

$$L_{(\theta_1 \xi)} = (\tilde{R}^4 + q^2)^{\frac{1}{2}} \sin \theta_1, \quad (49)$$

which simplifies the u -equation as follows,

$$u^3 ((\tilde{R}^4 + q^2)^{\frac{1}{2}} - q) \sin \theta_1 = 0. \quad (50)$$

It has a solution only for $u = 0$ or $\theta_1 = 0$, which signifies that the only non-generate solution is for minimal case. In case (ii),

$$L_{(\theta_1 \xi)} = q \sin \theta_1 \quad (51)$$

So the u -equation is automatically satisfied. So there exists a solution with non-zero flux q for any u_0 . We use kappa symmetry projections to check the supersymmetry preserved by these embeddings in the next section. It should be mentioned that kappa symmetry gauge fixing of the $D4$ brane embeddings considered in this paper from the world volume point of view is a subject which requires further study. For instance, for the case of fundamental strings and D2 branes wrapping $AdS_2 \times S^1$ and at the minkowski boundary of AdS_4 interesting gauges were discussed in [16].

III. SUPERSYMMETRY OF EMBEDDINGS

In this section, we find BPS configurations of D-branes that are wrapped on compact portions of our background, and are point like in the AdS. In order for the D-brane to be supersymmetric, we only need to check that the kappa-symmetry conditions [46–51]

$$\Gamma \epsilon = \epsilon. \quad (52)$$

is satisfied, where ϵ is the Killing spinor corresponding to the unbroken supersymmetry. The projection matrix is given by

$$d^{p+1} \eta \Gamma = -e^{-\Phi} \mathcal{L}_{DBI}^{-1} e^{\mathcal{F}} \wedge X|_{vol}, \quad (53)$$

with $X = \bigoplus_n \Gamma_{(2n+1)} \Gamma_{11}^{n+1} \mathbb{1}$ and \mathcal{L}_{DBI}^{-1} is to be evaluated on the background. Also, $\Gamma_{(n)} = \frac{1}{n!} d\eta^{i_n} \wedge \dots \wedge d\eta^{i_1} \Gamma_{i_1 \dots i_n}$ where $\Gamma_{i_1 \dots i_n}$ is the pullback for the target space gamma matrices $\Gamma_{i_1 \dots i_n} = \partial_{i_1} X^{m_1} \dots \partial_{i_n} X^{m_n} \Gamma_{m_1 \dots m_n}$. Γ has the special property that it squares to one and is traceless. It follows that one can use Γ to project out half of the worldvolume fermions, thus preserving supersymmetry. In type IIA case, the two 16-component MajoranaWeyl spinorial coordinates of opposite chirality form a 32-component Majorana spinor ψ^α , $\alpha = 1, \dots, 32$. In the Weyl basis, the corresponding gamma matrices are given by real block-off-diagonal matrices with a diagonal Γ_{11} -matrix. To find the relevant Killing spinor equation for this background we look at the supersymmetry transformation of the gravitino

$$\delta\Psi_\mu = D_\mu\epsilon - \frac{1}{288} \left(\Gamma_\mu^{\nu\lambda\rho\sigma} - 8\delta_\mu^\nu \Gamma^{\lambda\rho\sigma} \right) F_{\nu\lambda\rho\sigma} \epsilon, \quad D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon. \quad (54)$$

The 4-form corresponding to the $AdS_4 \times S^7$ solution is $F_{\nu\lambda\rho\sigma} = 6 \varepsilon_{\nu\lambda\rho\sigma}$, where the epsilon symbol is the volume form on AdS_4 (so the indices take the values 0, 1, 2, 3). Plugging this into the variation above one finds the Killing spinor equation

$$D_\mu\epsilon = \frac{1}{2} \hat{\gamma} \gamma_\mu \epsilon \quad (55)$$

where μ runs over all 11 coordinates, and $\hat{\gamma} = \gamma^{0123}$. Calculating the spin-connection from $\omega_\mu^{\hat{a}\hat{b}} = e_{\hat{a}}^\mu (\partial_\mu e^{\hat{b}} + e^{\tau\hat{b}} \Gamma_{\tau\mu}^\nu)$ we find for the AdS_4 ,

$$\omega_u^{\hat{u}\hat{u}} = 0, \quad \omega_0^{\hat{u}\hat{0}} = -u dx_0, \quad \omega_1^{\hat{u}\hat{1}} = -u dx_1, \quad (56)$$

$$\omega_2^{\hat{u}\hat{2}} = -u dx_2, \quad (57)$$

admits a full compliment of thirty-two independent solutions. We denote by $\gamma_a = e_a^m \Gamma_m$ the tangent space gamma matrices.

To proceed, here we will construct the Killing spinors preserved in the $AdS_4 \times \mathcal{CP}^3$ background starting from $AdS_4 \times S^7$ background. Some relevant calculations have been performed in [37, 52, 53]. We however repeat the calculations in our basis and also introduce projection operators in this basis. We take the following form for the AdS_4

$$ds_{AdS_4}^2 = \frac{du^2}{u^2} + u^2 (dx \cdot dx)_3 \quad (58)$$

$$= \frac{du^2}{u^2} + u^2 [-(dx^0)^2 + (dx^1)^2 + (dx^2)^2] \quad (59)$$

The three-dimensional $N = 6$ CS theory is conjectured to be dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$. To understand the action of the \mathbb{Z}_k orbifold, it is instructive to write the S^7 as a circle fibration over \mathcal{CP}^3 , where the orbifold acts on the fiber. For large k the radius of this ‘‘M-theory circle’’ becomes small, so the theory can be described in terms of type IIA string theory on $AdS_4 \times \mathcal{CP}^3$ with the metric

$$ds^2 = \frac{R^3}{4k} (ds_{AdS_4}^2 + 4ds_{\mathcal{CP}^3}^2). \quad (60)$$

The metric on \mathcal{CP}^3 can be written in terms of four complex projective coordinates z_i as

$$ds_{\mathcal{CP}^3}^2 = \frac{1}{\rho^2} \sum_{i=1}^4 dz_i d\bar{z}_i - \frac{1}{\rho^4} \left| \sum_{i=1}^4 z_i d\bar{z}_i \right|^2, \quad \rho^2 = \sum_{i=1}^4 |z_i|^2. \quad (61)$$

A specific representations in terms of angular coordinates is obtained by parameterizing S^7/Z_k as [39, 42], $z_1 = \cos \frac{\xi}{2} \cos \frac{\vartheta_1}{2} e^{i(2\varphi_1+\psi+\zeta)/4}$, $z_2 = \cos \frac{\xi}{2} \sin \frac{\vartheta_1}{2} e^{i(-2\varphi_1+\psi+\zeta)/4}$, $z_3 = \sin \frac{\xi}{2} \cos \frac{\vartheta_2}{2} e^{i(2\varphi_2-\psi+\zeta)/4}$, $z_4 = \sin \frac{\xi}{2} \sin \frac{\vartheta_2}{2} e^{i(-2\varphi_2-\psi+\zeta)/4}$. The metric on S^7 is then given by

$$ds_{S^7}^2 = \frac{1}{4} \left[d\xi^2 + \cos^2 \frac{\xi}{2} (d\vartheta_1^2 + \sin^2 \vartheta_1^2 d\varphi_1^2) + \sin^2 \frac{\xi}{2} (d\vartheta_2^2 + \sin^2 \vartheta_2^2 d\varphi_2^2) \right. \\ \left. + \sin^2 \frac{\xi}{2} \cos^2 \frac{\xi}{2} (d\psi + \cos \vartheta_1 d\varphi_1 - \cos \vartheta_2 d\varphi_2)^2 + \frac{1}{4} (d\zeta + A)^2 \right], \quad (62)$$

$$A = \cos \xi d\psi + 2 \cos^2 \frac{\alpha}{2} \cos \vartheta_1 d\varphi_1 + 2 \sin^2 \frac{\xi}{2} \cos \vartheta_2 d\varphi_2.$$

The angle ζ appears only in the last term and if we drop it we end up with the metric on \mathcal{CP}^3 ,

$$ds_{\mathcal{CP}^3}^2 = \frac{1}{4} \left[d\xi^2 + \cos^2 \frac{\xi}{2} (d\vartheta_1^2 + \sin^2 \vartheta_1^2 d\varphi_1^2) + \sin^2 \frac{\xi}{2} (d\vartheta_2^2 + \sin^2 \vartheta_2^2 d\varphi_2^2) \right. \\ \left. + \sin^2 \frac{\xi}{2} \cos^2 \frac{\xi}{2} (d\chi + \cos \vartheta_1 d\varphi_1 - \cos \vartheta_2 d\varphi_2)^2 \right]. \quad (63)$$

Although, the above metric is slightly different from the one of eqn. (5) used earlier in obtaining solutions, it can be seen later that it does not effect the analysis in this section.

The vielbeins coming from $g_{\mu\nu} = e_{\mu}^{\hat{a}} e_{\nu}^{\hat{a}} \eta_{\hat{a}\hat{b}}$ for AdS_4 are

$$e_u^{\hat{u}} = \frac{R}{2u} du, \quad e_0^{\hat{0}} = \frac{R}{2} u dx_0, \quad e_1^{\hat{1}} = \frac{Ru}{2} dx_1, \quad e_2^{\hat{2}} = \frac{Ru}{2} dx_2, \quad (64)$$

and for S^7 ,

$$e^4 = \frac{R}{2} d\xi, \quad e^5 = \frac{R}{2} \cos \frac{\xi}{2} d\vartheta_1, \quad e^6 = \frac{R}{2} \sin \frac{\xi}{2} d\vartheta_2, \\ e^7 = \frac{R}{2} \cos \frac{\xi}{2} \sin \frac{\xi}{2} (\cos \vartheta_1 d\varphi_1 - \cos \vartheta_2 d\varphi_2 + d\psi), \\ e^8 = \frac{R}{2} \cos \frac{\xi}{2} \sin \vartheta_1 d\varphi_1, \quad e^9 = \frac{R}{2} \sin \frac{\xi}{2} \sin \vartheta_2 d\varphi_2, \\ e^{\natural} = -\frac{R}{4} \left(d\zeta + 2 \cos^2 \frac{\xi}{2} \cos \vartheta_1 d\varphi_1 + 2 \sin^2 \frac{\xi}{2} \cos \vartheta_2 d\varphi_2 + \cos \xi d\psi \right). \quad (65)$$

Using the change of coordinates $u = e^r$, the killing spinor equation takes the form:

$$\partial_r \epsilon - \frac{1}{2} \hat{\gamma} \gamma_r \epsilon = 0, \quad \partial_p \epsilon + \frac{1}{2} \gamma_p (\hat{\gamma} + \gamma_r) \epsilon = 0, \quad (66)$$

where p runs over 0, 1, 2. The solution of these equations is:

$$\epsilon = e^{\frac{1}{2} r \hat{\gamma} \gamma_r} \left(1 + \frac{1}{2} \hat{\gamma} x^p \gamma_p (1 - \hat{\gamma} \gamma_r) \right) \epsilon_0, \quad (67)$$

where ϵ_0 is an arbitrary constant spinor. The full solution for the killing spinor also includes the function $h(\mathcal{CP}^3)$, which appears from solving the Killing equation on \mathcal{CP}^3 . A quicker way to obtain this, is the equation on the sphere given as

$$(D_\alpha + \frac{1}{2} \Gamma_\alpha) \epsilon = 0, \quad (68)$$

where D_α is the covariant derivative on S^7 , and the solution is given by

$$h(\mathcal{CP}^3) = e^{\frac{\xi_1}{4}(\hat{\gamma}\gamma_4 - \gamma_{74})} e^{\frac{\vartheta_1}{4}(\hat{\gamma}\gamma_5 - \gamma_{84})} e^{\frac{\vartheta_2}{4}(\gamma_{79} + \gamma_{46})} e^{-\frac{\xi_1}{2}\hat{\gamma}\gamma_4} e^{-\frac{\xi_2}{2}\gamma_{58}} e^{-\frac{\xi_3}{2}\gamma_{47}} e^{-\frac{\xi_4}{2}\gamma_{69}} \quad (69)$$

where the ξ_i are given by $\xi_1 = \frac{2\varphi_1 + \chi + \zeta}{4}$, $\xi_2 = \frac{-2\varphi_1 + \chi + \zeta}{4}$, $\xi_3 = \frac{2\varphi_2 - \chi + \zeta}{4}$, $\xi_4 = \frac{-2\varphi_2 - \chi + \zeta}{4}$. The Dirac matrices were chosen such that $\gamma_{01234567894} = 1$. Similar calculations in different coordinate systems were done in [43, 44]. To see which Killing spinors survive the orbifolding, one writes the spinor ϵ_0 in a basis which diagonalizes $i\hat{\gamma}\gamma_4\epsilon_0 = s_1\epsilon_0$, $i\gamma_{58}\epsilon_0 = s_2\epsilon_0$, $i\gamma_{47}\epsilon_0 = s_3\epsilon_0$, $i\gamma_{69}\epsilon_0 = s_4\epsilon_0$. All the s_i take values ± 1 and by our conventions on the product of all the Dirac matrices, the number of negative eigenvalues is even. Now consider a shift along the ζ circle, which changes all the angles by $\xi_i \rightarrow \xi_i + \delta/4$, the Killing spinors transform as

$$\mathcal{M}\epsilon_0 \rightarrow \mathcal{M}e^{i\frac{\delta}{8}(s_1 + s_2 + s_3 + s_4)}\epsilon_0. \quad (70)$$

This transformation is a symmetry of the Killing spinor when two of the s_i eigenvalues are positive and two negative and not when they all have the same sign (unless δ is an integer multiple of 4π). Note that on S^7 the radius of the ζ circle is 8π , so the \mathbb{Z}_k orbifold of S^7 is given by taking $\delta = 8\pi/k$. The allowed values of the s_i are therefore

$$(s_1, s_2, s_3, s_4) \in \left\{ \begin{aligned} &(+, +, -, -), (+, -, +, -), (+, -, -, +), \\ &(-, +, +, -), (-, +, -, +), (-, -, +, +) \end{aligned} \right\} \quad (71)$$

Each configuration represents four supercharges, so the orbifolding breaks $1/4$ of the supercharges (except for $k = 1, 2$) and leaves 24 unbroken supersymmetries.

The supersymmetry of the $AdS_3 \times CP^1$ brane embeddings discussed in the previous section is checked as follows. The explicit form of the kappa symmetry projection is

$$\epsilon = -\frac{1}{u^3(1+q^2)}\gamma^{01}\left((qu^3 - c)\gamma^2 + \sqrt{(u^6 + 2cqu^3 - c^2)}\gamma^3\right)(-\gamma_{584} + q)\epsilon. \quad (72)$$

As in [37] the projector Γ involving the flux on the worldvolume of the brane is considered for convenience. Preservation of supersymmetry requires that this condition must be satisfied for some subset of the background Killing spinors at all points on the brane worldvolume. In particular, it must hold at all values of $x^p = (x^0, x^1, x^2)$. Below, we explicitly calculate the supersymmetry preserved by the $c = 0$ embeddings and argue that $c \neq 0$ ones break all supersymmetry. Before proceeding, it is necessary to project the solutions for the killing spinors on to the brane world volume. This is done as follows. Using the solution in eqn. (33) and after a few manipulations, it is convenient to write the killing spinors as:

$$\epsilon = \left(e^{\frac{1}{2}r\hat{\gamma}\gamma_r} + \frac{1}{2}e^{\frac{1}{2}r\hat{\gamma}\gamma_r}(\gamma_r + \hat{\gamma})(x^l\gamma_l + \tilde{x}\gamma_2) - \frac{q}{2}e^{-r}(\gamma_r + \hat{\gamma})\gamma_2\right)h(\mathcal{CP}^3)\epsilon_0, \quad (73)$$

where $l = 0, 1$. From the terms in the Killing spinors which are linear in x^l we find the following condition:

$$\begin{aligned} &e^{\frac{1}{2}r\hat{\gamma}\gamma_r}(\gamma_r + \hat{\gamma})\gamma_l h(\mathcal{CP}^3)\epsilon_0 \\ &= -\frac{1}{(1+q^2)}\gamma_{01}\left(e^{-\frac{1}{2}r\hat{\gamma}\gamma_r}q(\gamma_3 - \gamma_{2584}) + e^{\frac{1}{2}r\hat{\gamma}\gamma_r}(-\gamma_{3584} + q^2\gamma_2)\right)(\gamma_r + \hat{\gamma})\gamma_l h(\mathcal{CP}^3)\epsilon_0. \end{aligned} \quad (74)$$

To proceed further, we start by noticing that terms in the function $h(\mathcal{CP}^3)$ in (69), which do not contain gamma matrices cancel generally. For the rest of the terms in $h(\mathcal{CP}^3)$, one checks that Γ commutes with $\hat{\gamma}\gamma_5$ and $\gamma_{8\sharp}$. Further, one can multiply by the inverse of $h(\mathcal{CP}^3)$ as,

$$h(\mathcal{CP}^3)^{-1}(1 - \Gamma)h(\mathcal{CP}^3)\epsilon_0 = 0, \quad (75)$$

to eliminate $h(\mathcal{CP}^3)$ completely from the equations. The eqn. (74) gives two sets of conditions:

$$\frac{1}{(1 + q^2)}\gamma_{01}(\gamma_3 + \gamma_{258\sharp})(\gamma_r + \hat{\gamma})\epsilon_0 = 0, \quad (76)$$

and

$$\frac{1}{(1 + q^2)}\gamma_{01}(\gamma_{358\sharp} + q^2\gamma_2)(\gamma_r + \hat{\gamma})\epsilon_0 = (\gamma_r + \hat{\gamma})\epsilon_0. \quad (77)$$

To solve eqns. (76) and (77), let us introduce the projections,

$$\gamma_{012}\epsilon_0 = \pm\epsilon_0^\pm. \quad (78)$$

Using eqn. (78), both the conditions (76) and (77) are solved by introducing the projection,

$$(1 + \gamma_{3258\sharp})\epsilon_0^+ = 0 \quad (79)$$

where, ϵ_0^- disappears from the equations. Thus, if one introduces the projections $\gamma_{3258\sharp}\epsilon_0^\pm = \pm\eta_\pm$, then eqn. (79) means that η_+ is eliminated and η_- remains undetermined at this level. Eqn. (79) will be analyzed further below.

The remaining terms from (73) give the following kappa symmetry projection:

$$\begin{aligned} & \left[e^{\frac{1}{2}r\hat{\gamma}\gamma_r}(1 + \frac{1}{2}\tilde{x}(\gamma_r + \hat{\gamma})\gamma_2) - e^{-\frac{1}{2}r\hat{\gamma}\gamma_r}\frac{q}{2}(\gamma_r + \hat{\gamma})\gamma_2 \right] h(\mathcal{CP}^3)\epsilon_0 \\ &= -\frac{1}{(1 + q^2)}\gamma_{01} \left[\{e^{-\frac{1}{2}r\hat{\gamma}\gamma_r}q(\gamma_3 - \gamma_{258\sharp}) + e^{\frac{1}{2}r\hat{\gamma}\gamma_r}(-\gamma_{358\sharp} + q^2\gamma_2)\}(1 + \frac{1}{2}\tilde{x}(\gamma_r + \hat{\gamma})\gamma_2) \right. \\ & \quad \left. - \{e^{\frac{1}{2}r\hat{\gamma}\gamma_r}q(\gamma_3 - \gamma_{258\sharp}) + e^{-\frac{1}{2}r\hat{\gamma}\gamma_r}(-\gamma_{358\sharp} + q^2\gamma_2)\}\frac{q}{2}(\gamma_r + \hat{\gamma})\gamma_2 \right] h(\mathcal{CP}^3)\epsilon_0. \end{aligned} \quad (80)$$

The kappa-symmetry projection condition (72) has to be satisfied at all points on the brane world-volume. Commuting $h(\mathcal{CP}^3)$ as before, thus eqn. (80) gives rise to two independent conditions,

$$\begin{aligned} & \epsilon_0^- + (1 + \tilde{x}\gamma_{32})\epsilon_0^+ \\ &= -\frac{1}{(1 + q^2)}\gamma_{01} \left((-\gamma_{358\sharp} + q^2\gamma_2)(\epsilon_0^- + (1 + \tilde{x}\gamma_{32})\epsilon_0^+) - q^2(\gamma_3 - \gamma_{258\sharp})\gamma_{32}\epsilon_0^+ \right) \end{aligned} \quad (81)$$

and

$$q\gamma_{32}\epsilon_0^+ = \frac{1}{(1 + q^2)}\gamma_{01} \left(q(\gamma_3 - \gamma_{258\sharp})(\epsilon_0^- + (1 + \tilde{x}\gamma_{32})\epsilon_0^+) - q(-\gamma_{358\sharp} + q^2\gamma_2)\gamma_{32}\epsilon_0^+ \right) \quad (82)$$

If one introduces the projections,

$$\gamma_{3258\sharp}\epsilon_0^\pm = \pm\lambda_\pm, \quad (83)$$

then both the eqns. (81) and (82) can be solved by using the relation,

$$2\lambda_- = -\tilde{x} \gamma_{32} \eta_- . \quad (84)$$

λ_+ is undetermined as well. Now, let us analyze the conditions (79) and (83) further. If $s_1 = s_2$, then

$$\gamma_{58\mathfrak{h}} \epsilon_0 = \hat{\gamma} \epsilon_0 , \quad (85)$$

turning the projection condition (79) in to,

$$\gamma_{01} \epsilon_0 = \epsilon_0 . \quad (86)$$

On the other hand, if $s_1 = -s_2$, the condition is,

$$\gamma_{01} \epsilon_0 = -\epsilon_0 . \quad (87)$$

Notice that $s_1 = s_2$ is satisfied by eight supersymmetries and they satisfy the projection in eqn. (86). When $s_1 = -s_2$, there are sixteen supersymmetries and they satisfy the projection conditions (87). Thus, one sees that twelve of the supersymmetries are preserved and half of the target space supersymmetry is broken by these embeddings. As in the $AdS_5 \times S^5$, the projections do not depend on the flux, but, on the value of \tilde{x} . Thus, each extra defect breaks the supersymmetry further.

For $c \neq 0$, one has different type of conditions. For instance, one ends up with conditions of the form:

$$-\frac{1}{u^3(1+q^2)} \gamma^{01} \left(-(qu^3 - c) \gamma^{258\mathfrak{h}} + q \sqrt{(u^6 + 2cqu^3 - c^2)} \gamma^3 \right) h(\mathcal{CP}^3) \epsilon_0 = 0 \quad (88)$$

From inspection, one sees that these conditions cannot hold at all points on the brane world volume. Thus, the $c \neq 0$ embeddings corresponding to asymptotically $AdS_3 \times S^2$ branes, break all supersymmetry.

For the embeddings corresponding to *case B*, the kappa-symmetry projection leads to:

$$\epsilon = -\gamma_{01} \left(\frac{\sqrt{2Cu^3 - C^2}}{u^3} \gamma_3 + (u^3 - C) \gamma_2 \right) \epsilon \quad (89)$$

Using the form of the killing spinors in eqn. (73), we end up with the following conditions:

$$\frac{\sqrt{2Cu^3 - C^2}}{u^3} \gamma_{013} h(\mathcal{CP}^3) \epsilon_0 = 0 , \quad (u^3 - C) \gamma_{012} h(\mathcal{CP}^3) \epsilon_0 = 0 \quad (90)$$

Clearly, these conditions cannot be satisfied for any non zero C and ϵ_0 . Thus, there are no non zero solutions to the kappa symmetry projections on the world volume and hence these embeddings break all supersymmetry as well, as in the cases considered in [37]. Even in the present case, one attribute the breaking of all supersymmetries to the misaligned branes in the $AdS_4 \times \mathcal{CP}^3$ background. For instance, if one had considered an embedding of the form given in eqn. (44), the induced metric in this case is very simple:

$$ds^2 = u^2(dx \cdot dx) . \quad (91)$$

In this case, the kappa symmetry projection is straightforward, giving:

$$\epsilon = -\text{sgn}(q) \gamma_{012} \epsilon. \quad (92)$$

Note that for the sign of Γ we have chosen, only embeddings which satisfy $(1 - \Gamma)\epsilon = 0$ respect supersymmetry and those with $(1 + \Gamma)\epsilon = 0$ break all supersymmetries. From eqn. (92), one can check that, for the embeddings with $q = +1$, the condition in eqn. (79) is enough and hence, half of the supersymmetry is preserved. The ones with $q = -1$ break all supersymmetry, as there is an over all negative sign, incompatible with our choice of Γ . Although, one expects that in the plane wave limit, both the signs of charges are compatible with the kappa symmetry projection. It would be interesting to further the analysis of this work to the plane wave limit and study the possible brane embeddings in that context.

IV. CONCLUSIONS

In this paper, we discussed various brane embeddings in $AdS_4 \times \mathcal{CP}^3$ and the supersymmetry preserved by them. In the case of duality between Type IIB string theory on $AdS_5 \times S^5$ and $N = 4$ super Yang-Mills system, there exists a good notion of renormalization group flow on the defect CFT, coming from the embedded branes. The defect theory should be associated with the boundary of the AdS_3 of the $AdS_3 \times S^2$ D4-brane. We expect such RG flows to hold for the present case as well, when the defect theory is conformal. As in the examples considered in [37], the D4-brane theory does not back react on the bulk. Thus, the deformations in which the boundary theory remains conformal but the defect theory runs are possible. An example is the asymptotically $AdS_3 \times S^2$ embeddings.

Since, the active scalar in our embeddings behaves at large u as:

$$ux \approx ux_0 + \frac{c}{4u^3}, \quad (93)$$

one expects that the scalar is dual to an operator O_x of conformal dimension 3 in the defect theory. Since, the operator O_x is sourced by the scalar x with vev c , it is expected to break supersymmetry.

Thus far in our discussion, the branes were considered to be probes. It would be interesting to consider the back reaction of the D4-brane in the target space geometry. In this case, one also has to find fully localized brane solutions in $AdS \times CP$ backgrounds. A fully localized solution is also important to understand the back reaction effects of the embeddings considered. So far we considered a single D4-brane, whose backreaction on the near-horizon geometry can be neglected in the 't Hooft limit, allowing it to be treated as a probe hosting open strings. Thus, it is not possible to get insight about the dynamics of boundary versus bulk modes in the CFT coming from gravity fluctuations. Further, intersecting brane solutions in $AdS \times CP$ backgrounds could be constructed to get insights in to the spectrum of defect conformal field theory. It would be interesting to construct the defect CFT and understand the correlation functions along the lines of [54]-[61]. Although we considered specific embeddings

in the $AdS_4 \times \mathcal{CP}^3$ background, it is possible to consider and classify other embeddings. The classification of embeddings can be done following the analysis in [37], using the number of intersection directions of Neumann and Dirichlet directions. Supersymmetry preserved by the brane configurations depends on the coordinate splitting. When they are sitting at arbitrary positions of the transverse space they preserve one quarter of supersymmetry, but they preserve twelve supercharges when located at the origin of the transverse space.

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APPENDIX A: D-BRANE FIELD EQUATIONS

In this appendix, we summarize the D-brane field equations derived in all generality and for all Dp-branes in [37]. The worldvolume action for a single Dp-brane is given by

$$\begin{aligned} I_p &= I_{DBI} + I_{WZ} \\ I_{DBI} &= -T_p \int_M d^{p+1} \eta e^{-\Phi} \sqrt{-\det(g_{ij} + \mathcal{F}_{ij})}, \quad I_{WZ} = T_p \int_M e^{\mathcal{F}} \wedge C, \end{aligned} \quad (A1)$$

with T_p the Dp-brane tension. Here η^i are the coordinates of the $(p+1)$ -dimensional worldvolume with metric $g_{ij} = g_{mn} \partial_i X^m \partial_j X^n$, following from the space-time string frame metric g_{mn} . Worldvolume field strength $\mathcal{F} = F - B$ is the gauge invariant two-form with $B_{ij} = \partial_i X^m \partial_j X^n B_{mn}$ the pullback of the target space NS-NS 2-form. Finally, we summarize the D-brane field equations

$$\begin{aligned} \partial_i (e^{-\Phi} \sqrt{-M} \theta^{ii_1}) &= \epsilon^{i_1 \dots i_{p+1}} \sum_{n \geq 0} \frac{1}{n! (2!)^n (q-1)!} (\mathcal{F})_{i_2 \dots i_{2n+1}}^n \bar{F}_{i_{2n+2} \dots i_{p+1}} \\ \sum_{n \geq 0} \frac{1}{n! (2!)^n q!} \epsilon^{i_1 \dots i_{p+1}} (\mathcal{F})_{i_1 \dots i_{2n}}^n \bar{F}_{i_{2n+1} \dots i_{p+1}} &= e^{-\Phi} \left(\sqrt{-M} (G^{ij} \partial_i X^p \partial_j X^n g_{mn} \Phi_{,p} - \Phi_{,m}) - \mathcal{K}_m \right). \end{aligned} \quad (A2)$$

where

$$\mathcal{K}_m = -\partial_i (\sqrt{-M} G^{ij}) \partial_j X^n g_{mn} - \sqrt{-M} M^{ij} \left((\partial_i \partial_j X^n) g_{mn} + \tilde{\Gamma}_{mnp} \partial_i X^n \partial_j X^p \right) \quad (A3)$$

$$\bar{F}_{m_1 \dots m_{q+1}} = f_{m_1 \dots m_{q+1}} - \frac{(q+1)!}{3!(q-2)!} H_{[m_1 \dots m_3} C_{m_4 \dots m_{q+1}]} \quad (A4)$$

where the following notations are introduced,

$$M_{ij} = (\partial_i X^m \partial_j X^n g_{mn} - \partial_i X^m \partial_j X^n B_{mn} + F_{ij}). \quad (A5)$$

and the inverse as M^{ij} such that $M^{ij}M_{jk} = \delta^i_k$. Moreover, $G^{ij} \equiv M^{(ij)}$ and $\theta^{ij} \equiv M^{[ij]}$. And we have $\tilde{\Gamma} = \Gamma - \frac{1}{2}H$, with Γ_{mnp} is the Levi-Civita connection of the target space metric and H_{mnp} is the field strength of the NS-NS two form. For the special case, when $F_{ij} = B_{mn} = \Phi = 0$, one has,

$$J^m = -\sqrt{-g}g^{ij}\mathcal{K}_{ij}^m \quad (\text{A6})$$

where

$$\mathcal{K}_{ij}^m = \gamma_{ij}^k \partial_k X^m - (\partial_i \partial_j X^m) - \Gamma_{np}^m \partial_i X^n \partial_j X^p \quad (\text{A7})$$

is the second fundamental form (γ_{ij}^k is the Levi-Civita connection of the induced worldvolume metric). If in addition $J_m = 0$, the field equation becomes

$$g^{ij}\mathcal{K}_{ij}^m = 0, \quad (\text{A8})$$

that is, the trace of the second fundamental form of the embedding must be zero.

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